Power System - I^{st}

Syllabus —

(1.) N/w matrices
(2.) Load flow studies
(3.) short circuit studies
(4.) Switch gear & protection
(5.) Power system stability
(6.) Economic depositor problem

Power system Analysis - I^{st} - Ramana

Network Matrices

Network matrices:-

For the Purpose of conductivity various kind of studies of P.S. We need the properties of a passive large size power system N/w.

→ Network matrices decides the properties of the N/w.

→ For analysing the system under steady state or dynamic state. The properties of the N/w is quite essential.

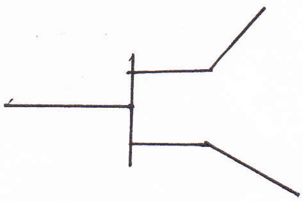
Frame of Reference:-

- Bus frame
- Branch frames
- Loop frames → $I_{Loop} = \frac{Y_{Loop}}{V_{Loop}}$

$$V_{Loop} = \frac{Z_{Loop}}{I_{Loop}}$$

V_i  V_k

$V_i - V_k$ → Branch voltage

Bus means electrical Junction 

$$I_{Bus} = Y_{Bus} \cdot V_{Bus}$$

$$V_{Bus} = Z_{Bus} \cdot I_{Bus}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Form Y-Bus :-

Leaving out the ref. bus ⓪ the no. of buses is 3.
∴ size of the y-bus is 3x3

The values marked as reactance into admittance matrix form.

$$Z \rightarrow Y$$

Ex → $j0.1 \longrightarrow -j10$

$-j10 \longrightarrow +j0.1$

$j0.2 \longrightarrow -j5$

$-j5 \longrightarrow j0.2$

$$j0.2 \longrightarrow \frac{10}{j0.2} = -j15$$

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Y_{11} - Total admittance connected to bus ①

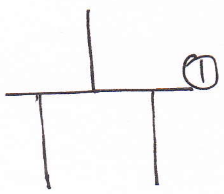
$= -j10 + j0.1 = -j.99$

$Y_{22} = +j0.1 - j5 = -j4.9$

$Y_{33} = -j5 + j0.2 = -j4.8$

Off diagonal element -

Y_{ik} - Negative value of series admittance connected b/w bus ① & ②



I_1 - Total incoming current to Bus

$\left.\begin{matrix} I_2 \\ I_3 \end{matrix}\right\}$ - Total outgoing current.

Bus Frames:-

Bus Admittance Matrix (YBus)

- Direct Inspection Method
- Singular transformation method

Bus impedance matrix (Z-Bus)

- $Y_{Bus} = Z^{-1}_{Bus}$
  $Z_{Bus} = Y^{-1}_{Bus}$
- Point by point method
  ZBus or building algorithm

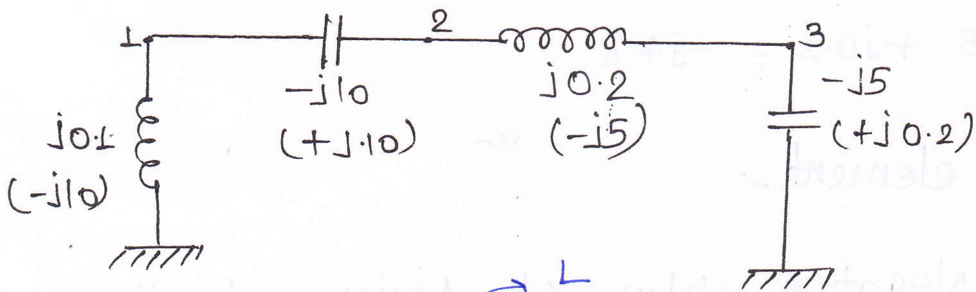
Primitive matrix

$$Y_{Bus} = [A^T][Y][A]$$

Non-Square matrix (singular matrix)

Y-Bus FORMATION :→

(a) Direct Inspection Method —



$Z = R \pm jX$ (L, C)

$Y = G \mp jB$ (L, C)

$Y_{12} = Y_{21} = -(+j\,0.1) = -j\,0.1$

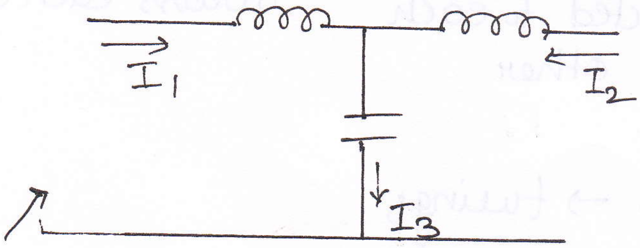
$Y_{13} = Y_{31} = -0 = 0$ (No direct connection b/w buss) 1 & 3

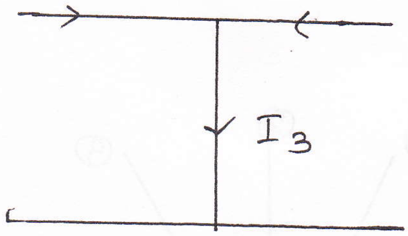
$Y_{23} = Y_{32} = -(-j5) = +j5$

Network Topology or Graph Theory →

Topology is the subject which deals with the geomatrical representation of the various objects.

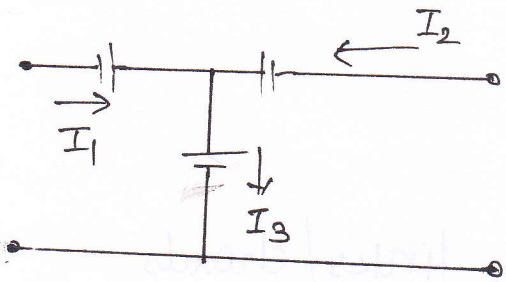
Geomatrical representation – form losses its identity not need to take origin a elements. Just make the graph e.g.





$I_3 = I_1 + I_2$ $\qquad\qquad$ $I_3 = I_1 + I_2$

$I_3 =$



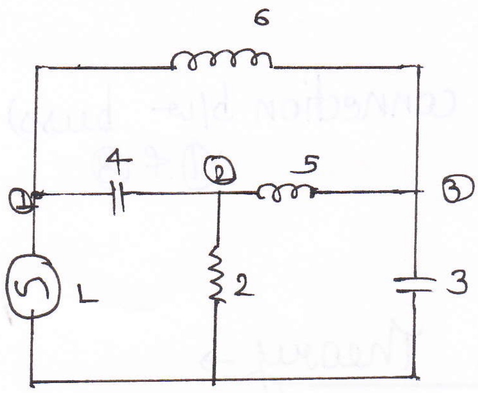
Use incidence matrix $\quad a_{ij} = \begin{array}{c} +1 \quad -1 \quad 0 \\ \begin{bmatrix} +1 & & \\ & & \end{bmatrix} \end{array}$

→ Network algorithm means finding the current through & voltage across every branch of a network.

→ Network analysis can be carried out of by using either loop analysis or node analysis

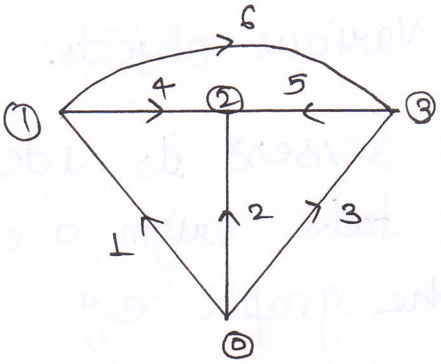
→ The basis for the nodal analysis is KCL and for loop analysis is KVL.

→ The KCL & KVL do not depend on type of element but depend upon the st^r of the N/W as graph of the N/W



Element $e = 6$

node $n = 4$

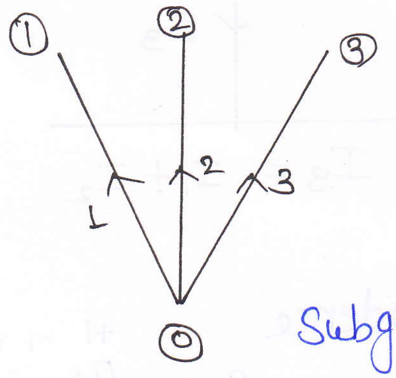


→ Main graph

→ Oriented graph

→ Planner Graph → because element across each othe

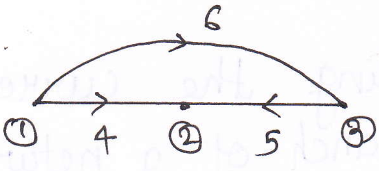
→ Well connected → All the node connected to each other



Subgraph

$1, 2, 3 \rightarrow$ twigs

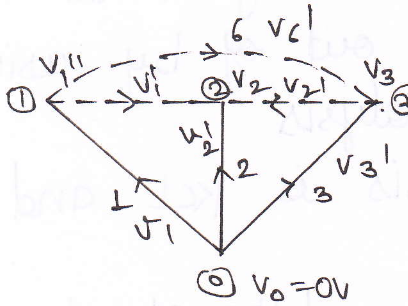
$t = n-1$



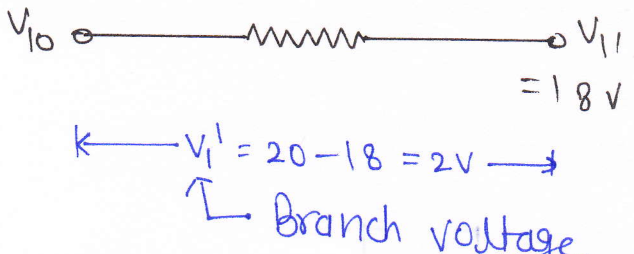
$4, 5, 6 \rightarrow$ links / chords

$l = e-(n-1)$

$= e-n+1$



V_0, V_1, V_2, V_3 — node voltage



$u_1 = V_0 - V_1 = -V_1$ $\quad$ $v_4 = V_1 - V_2$

$u_2 = V_0 - V_2 = -V_2$ $\quad$ $V_5 = V_3 - V_2$

$u_3 = V_0 - V_3 = -V_3$ $\quad$ $V_6 = V_1 - V_3$

Element node incidence matrix →

A = | → n-1 nodes
↓ element | a_{ij}

$a_{ij} = +1 \Rightarrow$ if the elements is incident away to the junction node and oriented away from junction node.

$-1 \Rightarrow$ Oriented forwards

$0 \Rightarrow$

A =

	1	2	3
1	-1	0	0
2	0	-1	0
3	0	0	-1
4	+1	-1	0
5	0	-1	+1
6	+1	0	-1

← By adding ref. bus the sum of coloumn is zero.

$$[v] = [A][V]$$

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$v_1 = -V_1$ $\qquad$ $v_4 = V_1 - V_2$

$v_2 = -V_2$ $\qquad$ $v_5 = -V_2 + V_3$

$v_3 = -V_3$ $\qquad$ $v_6 = V_1 - V_3$

Singular Transformation Method:-

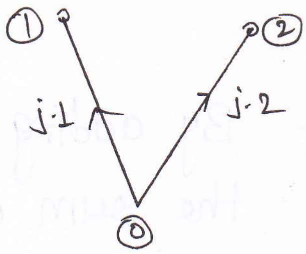
$$Y_{Bus} = [A^T][y][A]$$

$A \rightarrow$ element node incidence matrix

$[y] \rightarrow$ Pri admittance matrix $= [z^{-1}]$

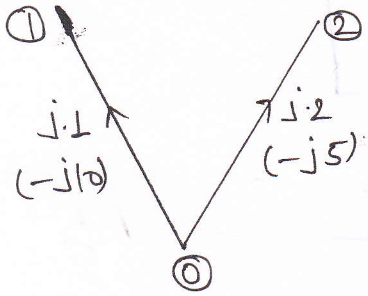
$[z] =$ Pri Impedance matrix

Exa:-



Form Y_{Bus}

(a) Direct inspection method :-



$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j10 & 0 \\ 0 & j5 \end{bmatrix}$$

Singular Transformation method :-

A =

b \ l	1	2
1	-1	0
2	0	-1

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Z =

e \ l	-1	-2
1	j0.1	0
2	0	j0.2

Z_{11}, Z_{22} - Self impedance

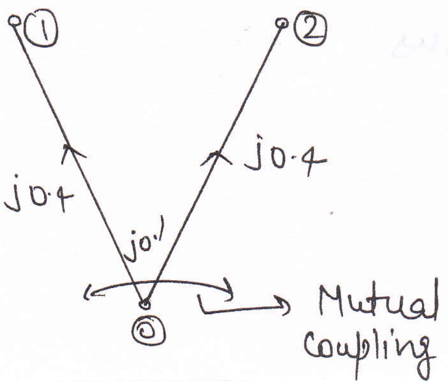
Z_{12}, Z_{21} - Mutual impedance

$$y = [Z^{-1}] = \begin{bmatrix} \frac{1}{j.1} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix}$$

$$Y_{Bus} = [A^T][y][A]$$

$$= \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix}$$

Exa -1



Form Y_{Bus}

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j\frac{10}{4} & -j10 \\ -j10 & -j\frac{10}{4} \end{bmatrix} \times \text{ (Not valid)}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad Z = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & j.4 & j0.1 \\ 2 & j0.1 & j0.4 \end{array}$$

$$y = z^{-1} = \frac{1}{AD-BC} \times \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

$$= \frac{1}{j0.4 \times j0.4 - j0.1 \times j0.1} \begin{bmatrix} j0.4 & -j0.1 \\ -j0.1 & j0.4 \end{bmatrix}$$

$$= \frac{1}{-0.15} \begin{bmatrix} j0.4 & -j0.1 \\ -j0.1 & j0.4 \end{bmatrix} = \begin{bmatrix} -j2.67 & +j0.667 \\ +j0.667 & -j2.67 \end{bmatrix}$$

$$Y_{Bus} = [A^T][y][A]$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -j2.67 & +j0.667 \\ +j0.667 & -j2.67 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -j2.67 & +j0.667 \\ +j0.667 & -j2.67 \end{bmatrix}$$

Z Bus Building Algorithm:-

$$Z_{Bus} = Y^{-1}_{Bus}$$

Disadvantage:-

(i) Finding the inverse of such a big matrix is very difficult.

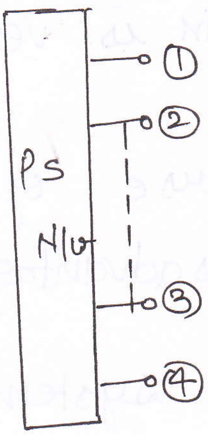
* Z-bus can be obtain from the inverse of Y-Bus. This method has following disadvantage.

(I) Owing to the no. of buses in power system the dimension of Y-bus is quite large finding the inverse of such a big matrix is always difficult.

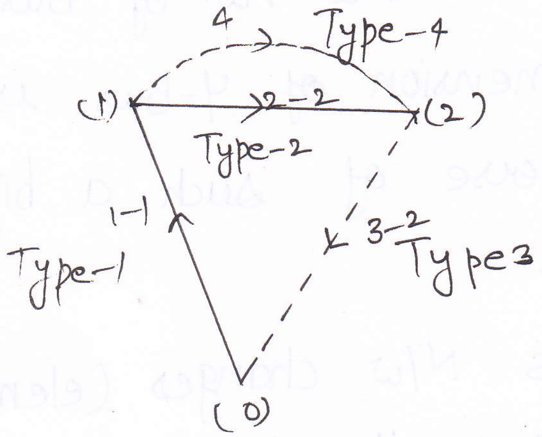
(II) In PS N/w changes (element addition deletion, and change the impedance value of element periodically takes place). When such changes takes place formation of the Z-bus from beginning through Y-bus is difficult.

To encounter the above difficulties the algorithm known as Zbus building algorithm is proposed.

Consider n-bus P.S. N/w →

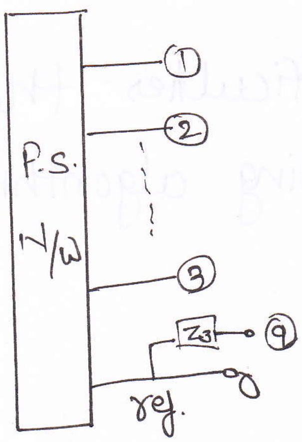


$$Z_{Bus,old} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{n1} & \cdots & \cdots & Z_{nn} \end{bmatrix}_{n \times n}$$



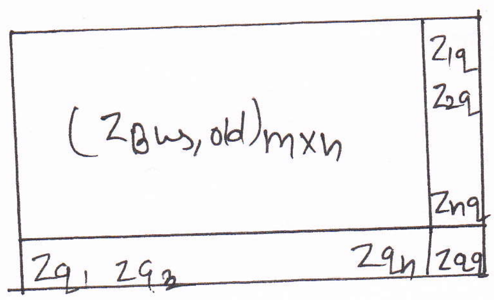
Type-1 Modification:-

An element will self impedence Z_s is added b/w ref bus (0) & new bus (q)



* When tewing existing then the new node will also exist.

In type-1 one row & one coloumn added due-to 1-2 imp exist.

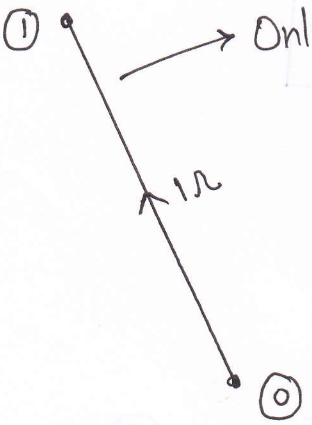


$$Z_{BUS,New} = \left[\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & & & & 0 \\ & & & & 0 \\ \hline 0 & 0 & 0 & 0 & Z_{qq} \end{array}\right]$$

☆ Off diagonal value will always 0 in type-1.

$$\boxed{Z_{qq} = Z_s}$$

Exp. 1.

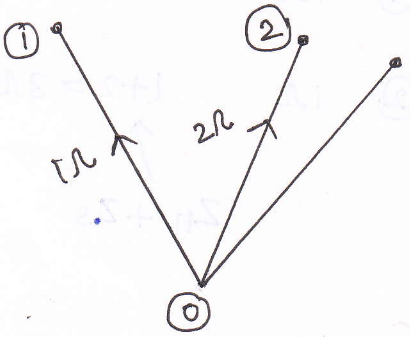


Only 1 bus connect
So $\boxed{q=1}$

$q=1$

$$\begin{array}{cc} & ① \\ Z_{BUS} = ① & \boxed{1\Omega} \end{array}$$

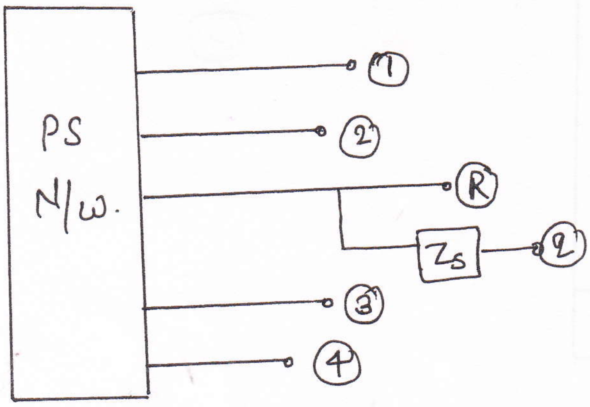
Exp. 2.



$$\begin{array}{ccc} & ① & ② \\ ① & Z_{11} = 1\Omega & 0 \\ ② & 0 & Z_{22} = 2\Omega \end{array}$$

☆ **Type-2. Modification :-**

An element will self impedance Z_s is added b/w already existing bus ⓚ & a new bus ⓠ.



$$Z_{BUS\,old} = \begin{bmatrix} Z_{11} & Z_{12} \cdots Z_{1K} \cdots Z_{1n} \\ Z_{21} & Z_{22} \cdots Z_{2K} \cdots Z_{2n} \\ \vdots & \\ Z_{K1} & Z_{K2} \cdots Z_{KK} \cdots Z_{Kn} \\ Z_{n1} & Z_{n2} \quad Z_{nK} \quad Z_{nn} \end{bmatrix}_{n}$$

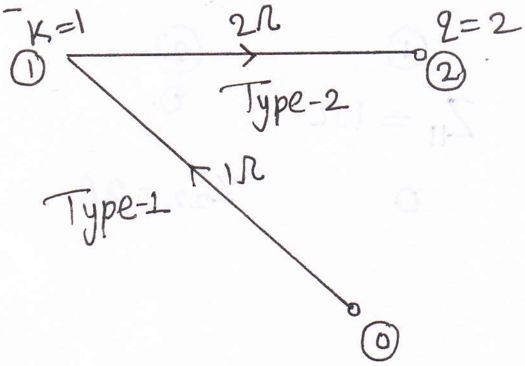
q^{th} colomn
copy q^{th} coloum
+ paste of q^{th} row

$$Z_{BUS,new} = \left[\begin{array}{cccc|c} Z_{11} & Z_{12} \cdots & Z_{1K} \cdots & Z_{1n} & Z_{1K} \\ \vdots & & & & Z_{2K} \\ Z_{K1} & Z_{K2} \cdots & Z_{KK} \cdots & Z_{Kn} & Z_{Kn} \\ & & & & \vdots \\ Z_{n1} & Z_{n2} \cdots & Z_{nK} \cdots & Z_{nn} & Z_{nK} \\ \hline Z_{K1} & Z_{K2} \cdots & Z_{KK} \cdots & Z_{Kn} & Z_{qq} \end{array}\right]$$

q^{th} Row

Copy K^{th} row & paste as q^{th} row.

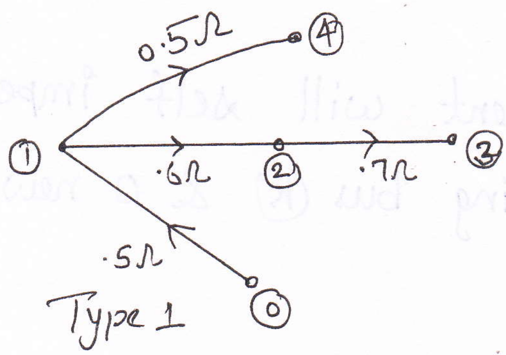
Exp. 1 :-



$$Z_{BUS} = \begin{array}{c|cc} & ① & ② \\ \hline ① & 1\Omega & 1\Omega \\ ② & 1\Omega & 1+2 = 3\Omega \end{array}$$

$Z_{11} + Z_s$

Exp. 2 :-



$$Z_{BUS} = \begin{array}{c|cccc} & ① & ② & ③ & ④ \\ \hline ① & .5 & .5 & & \\ ② & .5 & 1.1 & & \\ ③ & .7 & & & \\ ④ & .5 & & & \end{array}$$

$$\rightarrow Z_{BUS} = \begin{array}{c|c} & 1 \\ \hline 1 & \boxed{.5\Omega} \end{array}$$

$$\rightarrow Z_{BUS} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & .5\Omega & .5\Omega \\ 2 & .5\Omega & 1.1\Omega \end{array}$$

$$\rightarrow Z_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & .5 & .5 & .5 \\ 2 & .5 & 1.1 & 1.1 \\ 3 & .5 & 1.1 & 1.8 \end{array}$$

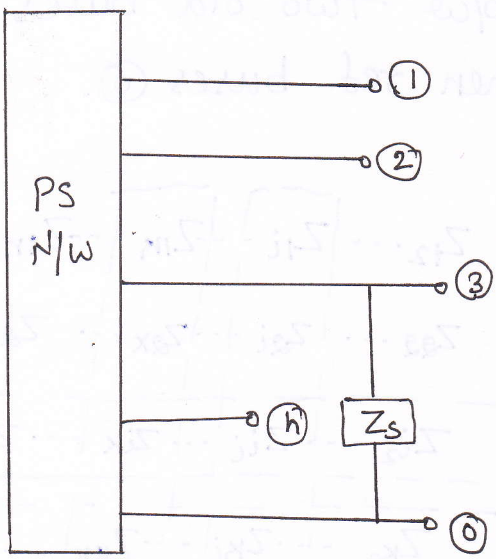
$$\rightarrow Z_{BUS} = \quad K=1 \quad \begin{array}{c|cccc} 1 & .5 & .5 & .5 & .5 \\ 2 & .5 & 1.1 & 1.1 & .5 \\ 3 & .5 & 1.1 & 1.8 & .5 \\ 4 & .5 & .5 & 0.5 & 1.3\Omega \end{array}$$

Paste   Paste

$Z_{11} + Z_s$

Type-3 Modification :-

An element will self impedance Z_s is added b/w already existing bus (K) & the ref. bus (0).



$$Z_{BUS,old} = \begin{bmatrix} Z_{11} & Z_{12} \cdots Z_{1K} \cdots Z_{1n} \\ Z_{21} & Z_{22} \cdots Z_{2K} \cdots Z_{2n} \\ \vdots & \\ Z_{K1} & Z_{K2} \cdots Z_{KK} \cdots Z_{Kn} \\ \vdots & \\ Z_{n1} & Z_{n2} \cdots Z_{nK} \cdots Z_{nn} \end{bmatrix}_n$$

K^{th} row

K^{th} coloum

$$Z_{BUS,new} = Z_{BUS,old} - \frac{1}{Z_{KK}+Z_s}\begin{bmatrix} Z_{1K} \\ Z_{2K} \\ \vdots \\ Z_{KK} \\ \vdots \\ Z_{nK} \end{bmatrix}\left[Z_{K1} Z_{K2} \cdots Z_{KK} Z_{Kn}\right]_n$$

Exp.

$Z_{BUS,old}$ =

	①	②
①	1Ω	1Ω
②	1Ω	3Ω

Now odd 3Ω resistance.
but K=2 & ⓪

$$Z_{BUS,new} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} - \frac{1}{3+3}\begin{bmatrix} 1 \\ 3 \end{bmatrix}[1, 3]$$

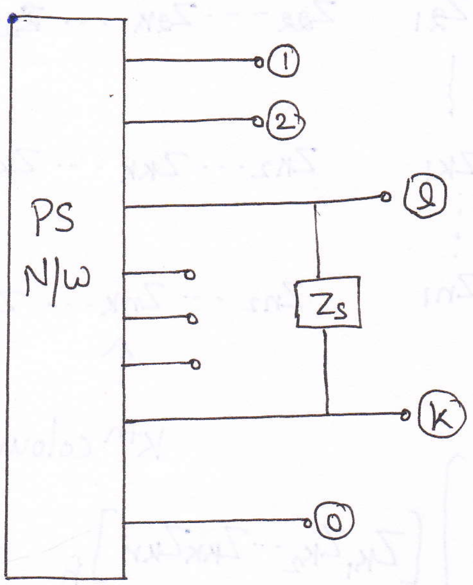
$$= \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 1/6 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 5/6 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$$

Type-4 Modification :-

An element with self impedance Z_s is added b/w two old buses ⓘ & ⓚ other then ref. buses ⓪.



$$Z_{BUS} = \begin{bmatrix} Z_{11} & Z_{12} \cdots & Z_{1i} & \cdots Z_{1k} & \cdots Z_{1n} \\ Z_{21} & Z_{22} \cdots & Z_{2i} & \cdots Z_{2k} & \cdots Z_{2n} \\ Z_{i1} & Z_{i2} \cdots & Z_{ii} & \cdots Z_{ik} & \cdots Z_{in} \\ Z_{k1} & Z_{k2} \cdots & Z_{ki} & \cdots Z_{kk} & \cdots Z_{kn} \\ Z_{n1} & Z_{n2} & Z_{ni} & Z_{nk} & Z_{nn} \end{bmatrix}$$

i^{th} row

k^{th} row

i^{th} column

k^{th} column

$$Z_{BUS,new} = Z_{BUS,old} - \frac{1}{Z_s + Z_{kk} + Z_{ii} - 2Z_{ik}} \begin{bmatrix} Z_{1i} - Z_{1k} \\ Z_{2i} - Z_{2k} \\ \vdots \\ Z_{ii} - Z_{ik} \\ Z_{ki} - Z_{kk} \\ Z_{ni} - Z_{nk} \end{bmatrix} \begin{bmatrix} Z_{i_1} - Z_{k_1} & Z_{i_2} - Z_{k_2} & \cdots & Z_{in} - Z_{kn} \end{bmatrix}$$

From ith column substract kth column to write as column

From ith row substract kth row to write as a row.

Exp.

$$Z_{BUS,old} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 5/6 & 1/2 \\ 2 & 1/2 & 3/2 \end{array}$$

$i = 1$; $K = 2$

$Z_s = 4$

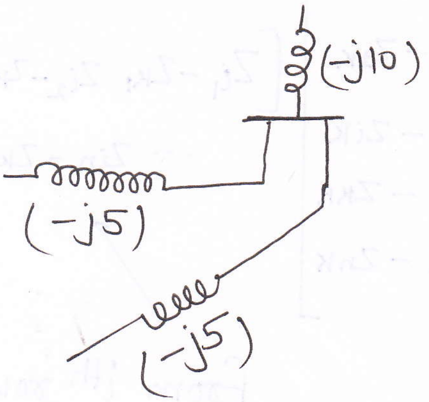
$$Z_{BUS,New} = \begin{bmatrix} 5/6 & 1/2 \\ 1/2 & 3/2 \end{bmatrix} - \frac{1}{4 + 3/2 + 5/6 - 2\times\frac{1}{2}} \begin{bmatrix} 1/3 \\ -1 \end{bmatrix} \begin{bmatrix} 1/3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5/6 & 1/2 \\ 1/2 & 3/2 \end{bmatrix} - \frac{3}{16} \begin{bmatrix} 1/9 & -1/3 \\ -1/3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5/6 & 1/2 \\ 1/2 & 3/2 \end{bmatrix} - \begin{bmatrix} 1/48 & -1/16 \\ -1/16 & 3/16 \end{bmatrix}$$

Chapter 3.

(2)



$$1/_{22} = -j10 - j5 - j5$$
$$= -j20$$

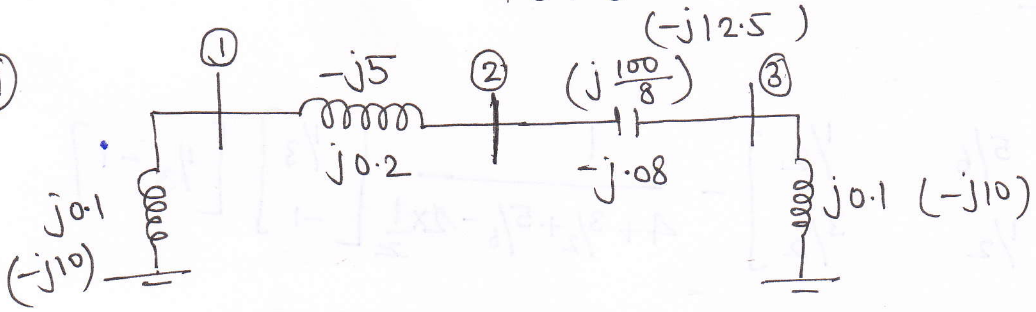
(6) $Y_{22} = -j10 - j10 + j0.05$

$= -j19.95$ P.U.

(7) $Y_{BUS} = Z^{-1}_{BUS} = \dfrac{1}{.9\times.6 - .2\times.2}\begin{bmatrix} .6 & .2 \\ -0.2 & 0.9 \end{bmatrix}$

$= 1.8$ P.U.

(11)



$$= j\begin{bmatrix} 15 & 5 & 0 \\ 5 & 7.5 & -12.5 \\ 0 & -12.5 & 2.5 \end{bmatrix}$$

P-52
30

$Z_s = j0.2\,\Omega$

Connected but bus(2) & ref.

Type-3 Modification :-

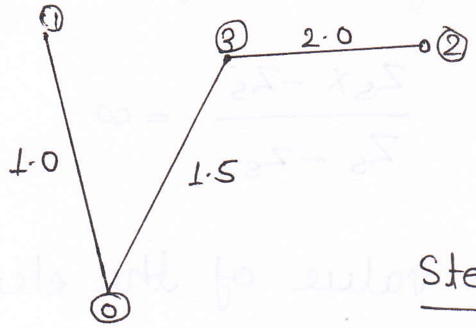
$K = 2$

$$Z_{BUS,new} = Z_{BUS,old} - \frac{1}{Z_{22}+Z_s}\begin{bmatrix} j.2860 \\ j.3408 \\ j.2586 \\ j.2414 \end{bmatrix}\begin{bmatrix} j0.2860 & j0.3408 \\ j.2586 & j.2414 \end{bmatrix}$$

$$Z_{22,new} = j.3408 - \frac{1}{j.3408 + j2} \times j.3408 \times j0.3408$$
$$= j.1260\Omega$$

$$Z_{23new} = j0.2586 - \frac{1}{j.3408 + j0.2} \times j.3408 \times j0.2586$$
$$= j0.095$$

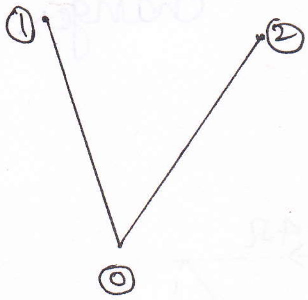
31.



add 1Ω -

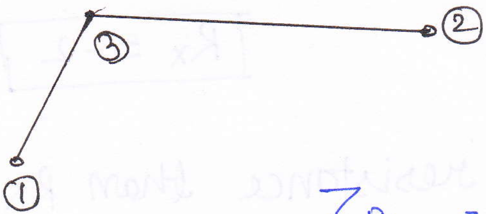
$$Z_{Bus} = \begin{array}{c|c} & 1 \\ 1 & 1\Omega \end{array}$$

Step - 2 add 1.5Ω



$$Z_{Bus} = \begin{array}{c|c|c} & 1 & 3 \\ \hline 1 & 1\Omega & 0 \\ 3 & 0 & 1.5\Omega \end{array}$$

Step-3 add 2Ω $K = 3$ $a = 2$



$$Z_{Bus} = \begin{array}{cccc} & 1 & 3 & 2 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 1.5 & 1.5 \\ 2 & 0 & 1.5 & 3.5 \end{array}$$

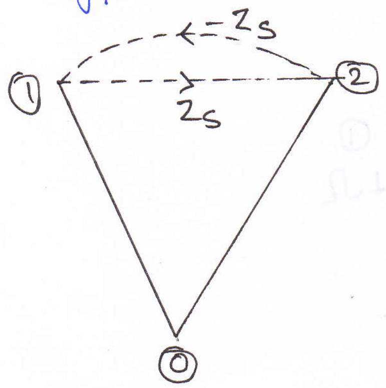
$$\boxed{Z_{22} = 3.5\Omega}$$

* Element Deletion:→

A self impedance Z_s connected b/w buses (1) & (k) bus to be removed say

→ The procedure is add $-Z_s$ across buses (1) & (k)

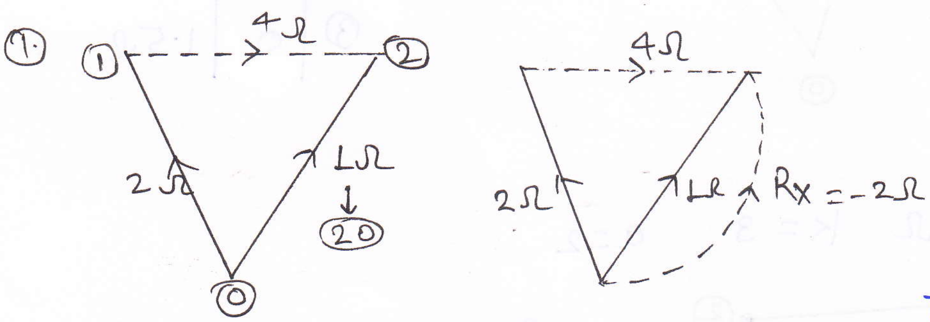
→ Type-3 or 4 modification may be required to perform



$$Z_s \| -Z_s = \frac{Z_s \times -Z_s}{Z_s - Z_s} = \infty$$

Impedence value of the element changes.

Exa:-



$$R_x \| 1\Omega = 2\Omega$$

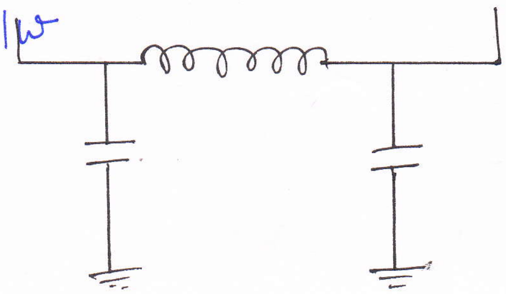
$$\frac{R_x}{1+R_x} = 2$$

$$\boxed{R_x = -2}$$

** If increase effective resistance than R_x should negative, If take decrease resistance then R_x should positive.

Ex- Pre que ↑ R from 1Ω to 2Ω

* In load flow the N/w always represent in π N/w not in T N/w



Per Unit Method

→ A per unit method uses per unit (P.u) values

→ A per unit value is a unit less quantity.

→ $$\text{P.u. value} = \frac{\text{Actual value in same unit}}{\text{Base or ref. value in same unit}}$$

Ex:- $V_a = 400kV$

$V_b = 100kV$

$$V_{pu} = \frac{400kV}{100kV} = 4\,P.u$$

$$\boxed{1\,P.u\ \text{voltage} = 100kV}$$

Ex- $V_a = 400kV$

$V_b = 400kV$

$$V_{pu} = \frac{400kV}{400kV} = 1\,Pu$$

$$\boxed{1\,P.u\ \text{voltage} = 400kV}$$

$$\boxed{\text{P.u value} \times 100 = \%\ \text{value}}$$

Advantage of P.u. method -

(1) It simplifies power system calculation.

(2) It avoids the discontinuity problem posed by the presence of T/F in P.s. N/w

Explanation →